Inflation targeting, anchoring of expectations, and the stability of liquidity trap[1]

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Abstract

This paper explores the expectational stability of a liquidity trap. We assume that the central bank announces an inflation target that anchors agents' expectations at the target. We finds that a liquidity trap steady state can be expectationally stable in the presence of inflation targeting. A targeted steady state is stable under learning regardless of the degree of central bank credibility, while the liquidity trap steady state becomes stable if the central bank is highly credible and the natural rate of interest is negative. In the stable liquidity trap, the economy exhibits an upward bias in inflation expectations and heterogeneous inflation expectations. These findings are consistent with empirical evidence.

JEL classification: C62; D82; D83; E32; E52

Keywords: Liquidity trap; Learning; Expectational stability; Inflation target; Central bank credibility

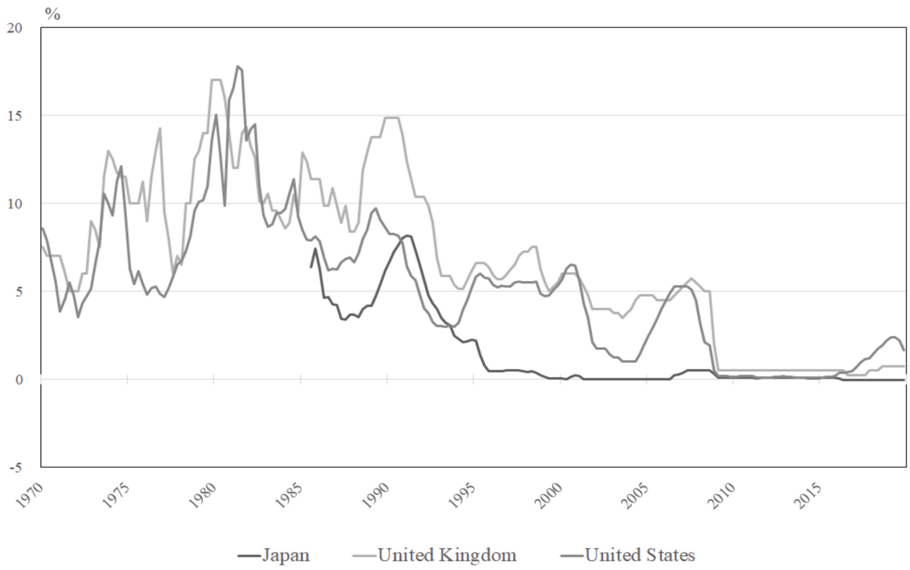

Figure 1: Inflation rates

1 Introduction

Since the global financial crisis in the late 2000s, developed economies have experienced a prolonged period of deflationary processes (Figure 1). To raise inflation expectations, central banks have drastically reduced nominal interest rates (Figure 2), and further introduced inflation targeting and unconventional monetary policies. However, until the late 2010s, economies fell into a so-called *liquidity trap*, where nominal interest rates were lowered to the zero lower bound, rendering conventional monetary policy ineffective in stimulating economies. While the U.S. and European economies have recently escaped from the liquidity trap, the Japanese economy has been trapped and stagnating since the bubble burst in the early 1990s (see Bullard, 2010; Aruoba et al., 2018).

The mechanism of a liquidity trap has been extensively studied. Benhabib et al. (2001) show that under the Taylor type of nominal interest rate rules, there exist two steady states: an intended, targeted steady state where the inflation rate achieves the target and an unintended, low steady state where the nominal interest rate is at the zero lower bound and the inflation rate is below the target. The latter steady state is used to explain the recent persistent stagnation fixed at the zero lower bound. Bullard (2010) considers that the Japanese economy has been stuck in the low steady state and cautioned that the U. S. economy could fall into the same situation. This framework is standard for examining the dynamics of the economy in recent stagnation fixed at the zero lower bound (e.g., Eggertsson and Woodford, 2003; Schmitt-Grohé and Uribe, 2017; Eggertsson et al., 2019). Related studies examine the effect of fiscal policies in a liquidity trap (Mertens and Ravn, 2014; Aruoba et al., 2018) and the effect of neo-Fisherian policies (Bilbiie, 2022; Uribe, 2022).

However, the low steady state is found to be unstable in plausible frameworks of expectation formation. McCallum (2001) linearizes Benhabib et al. (2001)'s model and finds that the low steady state is not stable under adaptive learning. This finding is complemented by Evans and Honkapohja (2005) and Evans et al. (2008), who show the global instability of the low steady state in the original nonlinear model. Further, this suggests a deflationary spiral in which the economy falls unboundedly below the low steady state. Evans and Honkapohja (2005) and Evans et al. (2008) obtain the same result under infinite horizon learning. Eusepi (2010) shows its robustness to communication between the central bank and the private sector, and Hommes and Lustenhouwer (2019a,b) show its robustness to the imperfect credibility of inflation targeting. These characteristics of the low steady state are inconsistent with recent evidence that developed economies have remained near the zero lower bound for several years. In particular, the Japanese inflation rate has remained stable around zero despite the nominal interest rate adhering to the zero lower bound over the past two decades (see Veirman, 2009; Gorodnichenko and Sergeyev, 2021).

This paper uses a standard linear New Keynesian (NK) model with a zero lower bound to re-examine the stability of the low steady state under adaptive learning. Following Orphanides and Williams (2005), we assume that the central bank announces an inflation target that anchors agents' expectations to the target. We examine whether the low steady state is expectationally stable

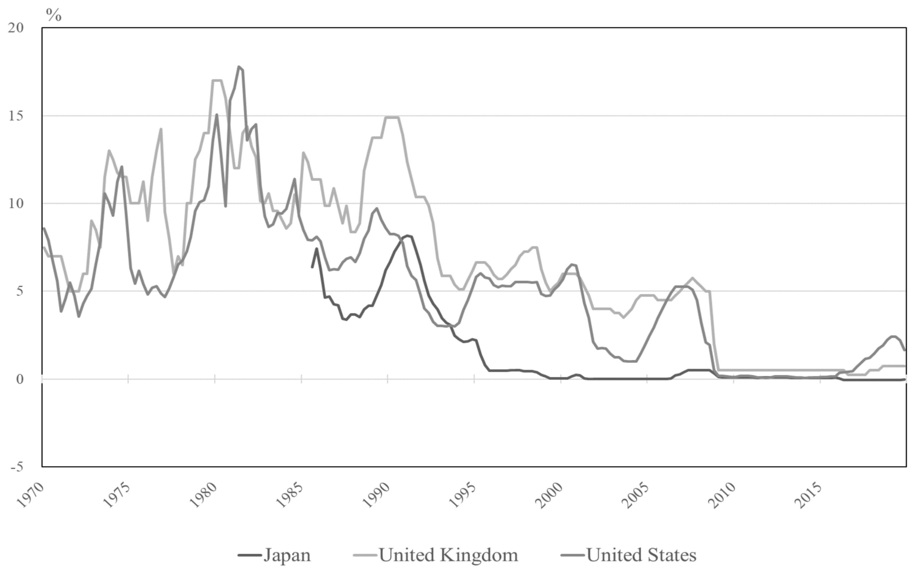

Figure 2: Policy rates

in the presence of inflation targeting. We also consider the case where the central bank (or inflation targeting) is imperfectly credible such that agents' expectations are imperfectly anchored. We then examine how the stability of the low steady state is affected by the degree of central bank credibility. In addition, we clarify the characteristics of the liquidity trap steady state in the presence of inflation targeting. If so, we investigate the characteristics of the stable low steady state.

Adaptive learning models have been confirmed to well explain the dynamics of recent data on inflation persistence and inflation expectations. It is often argued that expectation formation deviates from rational expectations. Candia et al. (2023) find that U. S. firms' expectation formation follows adaptive learning rather than rational expectations. Carvalho et al. (2023) find that adaptive learning models offer a more robust explanation of the dynamics of long-term inflation expectations indicators. Crump et al. (2023) find that the term structure of the U.S. interest rate is well explained by adaptive learning. Eusepi and Preston (2011) demonstrate that learning dynamics can provide an alternative explanation for business cycle fluctuations. Malmendier and Nagel (2016) build on adaptive learning and find that a weighted average of individuals' inflation experiences over their lifetimes strongly predicts individual inflation expectations.

Such expectation formation is believed to be significantly affected by inflation targeting. Inflation targeting announces to the public the central bank's objective and plan by setting a target for the long-term inflation rate. Since the 1970s, there has been consensus that the long-term inflation rate is primarily determined by the central bank (see Bernanke et al., 1999). This view was reinforced in the early 1980s when Paul Volcker's aggressive stance lowered the inflation rate from 10% to 4% (see Erceg and Levin, 2003). This led central banks to announce inflation targets, either implicitly or explicitly, and since the 1990s, inflation rates have stabilized around the targets (Figure 1). Even if the central bank is not sufficiently credible, inflation targeting is expected to affect the expectations formation in each period by anchoring long-term expectations.[1]

This paper finds that the low steady state can be expectationally stable in the presence of inflation targeting. The targeted steady state is always stable under learning regardless of the degree of central bank credibility, while the low steady state becomes stable if the central bank is highly credible and the natural rate of interest is negative. In the stable steady state, the economy exhibits an upward bias in inflation expectations and heterogeneous inflation expectations.

The evidence of the negative natural rate of interest is well provided in the literature, in particular, by Krugman (1998, 2000) for discussion about the Japanese deflation in the late 1990s and Summers (2014, 2015, 2016) for discussion about the U. S. and European "secular stagnation" in the early 2010s. Eggertsson and Woodford (2003) have proposed the "forward guidance" for a decline in the natural rate of interest so significant as to make nominal interest

[1] Nakagawa (2022) shows equilibrium with an interest-rate peg rule can be stable by inflation targeting, under which the steady state inflation rate is announced as a long-run inflation target.

rates hit the zero lower bound. Their empirical evidence is also well provided by Krugman (1998, 2000) and Eggertsson et al. (2019) for the U. S. economy, Brand et al. (2018) for the Euro economy, and Okazaki and Sudo (2018) for the Japanese economy. Our results provide theoretical support for these arguments in that the liquidity trap is associated with the negative natural rate in learning frameworks.

This paper is closely related to the literature on the stability of the liquidity trap steady state. To solve this puzzle, several recent studies have shown the conditions under which the low steady state becomes stable. Arifovic et al. (2018) show that the steady state can become stable by introducing social learning expectation formation. Lustenhouwer (2021) shows that pessimistic expectation formation in the private sector is a condition for stability. Eusepi (2007) shows that a monetary policy rule that responds to an expectations variable can be a condition. He shows that a liquidity trap SS (and sunspot equilibria near the trap) can be stable in a nonlinear NK model. The conditions are, however, that a monetary policy rule is forward-looking and the utility function of consumption and money is not separable, or that a production function with money exists. These are rather limited conditions, and the relationship between inflation targeting and the natural rate of interest is not well addressed. So far, little work has been done to explain the mechanism of the long-run liquidity trap in the Japanese economy.

The remainder of this paper is structured as follows. The next section presents our model. Section 3 examines whether the targeted steady state is stable in the presence of inflation targeting. Section 4 analyzes the stability of the low steady state under inflation targeting and clarifies the characteristics of the stable liquidity trap. Section 5 simulates recursive least-squares estimations of the liquidity trap. The final section concludes the paper.

2 Model

2.1 NK model

We use a standard New Keynesian model based on Evans and McGough (2018) and Hommes and Lustenhouwer (2019a):

$$x_t = -\alpha \left(i_t - E_t^* \pi_{t+1} - r\right) + E_t^* x_{t+1} + e_t, \tag{1}$$

$$\pi_t = \kappa x_t + \beta E_t^* \pi_{t+1} + u_t, \tag{2}$$

$$i_t = \max \left\{ i^I + \phi_\pi \left(\pi_t - \pi^I\right), i^U \right\}. \tag{3}$$

Endogenous variables x_t, π_t, and i_t represent the output gap, inflation rate, and nominal interest rate, respectively. Eq. (1) is a log-linearized intertemporal Euler equation derived from households' optimal choice of consumption. The parameter r represents the natural rate. Eq. (2) is the Phillips curve derived from the optimizing behavior of monopolistically competitive firms with Calvo price setting. The exogenous variables e_t and u_t are demand and supply shocks, respectively, and we assume that they follow independent and identically distributed (iid) processes for simplicity. Eq. (3) is the Taylor-type nominal interest rate rule with an effective lower bound i^U. If $i_t \geq 0$, the central bank responds to the deviation of π_t from the inflation target π^I by following the Taylor principle ($\phi_\pi > 1$), and parameter $i^I = r + \pi^I$ is the nominal interest rate corresponding to the inflation target. If $i_t < 0$, the central bank fixes i_t at the lower bound. E_t^* is the operator of agents' expectations at time t, which may or may not be rational. $\alpha > 0$, $\kappa > 0$, and $0 < \beta < 1$ are assumed.

In this paper, we consider the possibility that the natural rate can be negative. In standard NK models, the natural rate is equal to the inverse of the discount factor β, which cannot be negative. On the other hand, Eggertsson et al. (2019) consider the possibility of a negative natural rate in OLG models. Cochrane (2018) simulates shocks that make the natural rate negative. Schmitt-Grohé and Uribe (2017) assume confidence shocks where the natural rate is negative. For simplicity of our analysis, we abstract specific cases of the negative rate and simply assume the possibility of $r < 0$.

Under rational expectations ($E_t^* = E_t$), the fundamental rational expectations equilibrium (REE) takes the following form:

$$y_t = a + c w_t, \tag{4}$$

where $y_t \equiv (x_t, \pi_t)'$ and $w_t \equiv (e_t, u_t)'$, and a is the vector of constant terms and c is the 2×2 matrix of coefficients for w_t.

There are two steady states. The first one is the *intended, targeted steady state* that the central bank pursues when the effective lower bound i^U does not bind:

$$x^I = \left(\frac{1-\beta}{\kappa}\right)\pi^I, \quad \pi = \pi^I, \quad i = i^I \left(= r + \pi^I\right). \tag{5}$$

This is found to be stable under learning in the literature.

The other one is the *unintended, low steady state* that is realized when the lower bound binds:

$$x^U = -\left(\frac{1-\beta}{\kappa}\right)\left(r - i^U\right), \quad \pi^U = -\left(r - i^U\right), \quad i = i^U. \tag{6}$$

The literature shows that the low steady state is unstable under learning and the economy goes to a deflationary spiral (see Evans and McGough, 2018).

This steady state exists if the effective lower bound i^U binds at this steady state.

$$i^I + \phi_\pi \left(\pi - \pi^I\right) \geq i^U, \tag{7}$$

The low steady state exists if this condition is violated at this steady state. As $\phi_\pi > 1$, both conditions are reduced to the same condition:

Proposition 1 *Under rational expectations, the targeted and low steady states (5), (6) exist if and only if*

$$r + \pi^I - i^U \geq 0. \tag{8}$$

Otherwise, there are no steady states so that the economy explodes unboundedly.

3 Targeted steady state

The central bank announces the inflation-output gap target $\left(x^I, \pi^I\right)$ and controls the nominal interest rate to achieve this target. We assume the imperfect credibility of the central bank in the way that agents of proportion $\lambda \in [0, 1)$ (Type 1) believe the inflation target and the other agents of proportion $1 - \lambda$ do not believe or know that the central bank pursues the target (Type 2). Parameter λ represents the degree of credibility (see Hommes and Lustenhouwer, 2019a; Ho et al., 2021). First, we examine the stability of the targeted steady state in the presence of imperfectly credible inflation targeting. Candia et al. (2022) find that firms often vary their short- and long-term inflation expectations simultaneously and to the same extent, suggesting that steady-state inflation expectations are not fixed to a target (consistent with Type 2).

3.1 Expectations Formation

Under learning, agents have a perceived law of motion (PLM) of form (4) and specify and estimate the forecasting model of this form using least-squares estimation with available data. Under imperfectly credible inflation targeting, the two types of agents specify different forecasting models.

Type 1 agents recognize the steady state inflation rate and output gap as $\left(x^I, \pi^I\right)$, respectively, by the central bank's announcement of inflation targeting.[2] Then, they specify and estimate the following forecasting model:

$$y_t = a_1 + c_1 w_t,$$

where $a_1 \equiv \left(x^I, \pi^I\right)'$, c_1 is a 2×2 matrix of coefficients for w_t. Type 1 fix the steady state parameter a_1 at the target $\left(x^I, \pi^I\right)$, estimate only the coefficient parameter c_1, and form their forecasts:

$$E^*_{1t} y_{t+1} = \left(x^I, \pi^I\right)',$$

where E^*_{1t} is the operator of the Type 1 agents' expectations at time t, which may or may not be rational. Note that we do not need Type 1 expectations in each period to coincide with the inflation target. If there are exogenous shocks following AR (1), $E^*_{1t} y_{t+1}$ deviates from the inflation target and fluctuates around it. However, these extensions do not change our conclusions. To simplify the discussion, we assume that Type 1 expectations in each period coincide with the inflation target.

Type 2 agents do not believe or know the inflation target, but specify and estimate the following forecasting model:

$$y_t = a_2 + c_2 w_t,$$

[2] Mehrotra and Yetman (2018) have shown that longer-term forecasts are better anchored than shorter-term forecasts.

where $a_2 \equiv (a_{2x}, a_{2\pi})'$, and c_2 is the 2×2 matrix of coefficients for w_t. In contrast to Type 1, Type 2 must estimate all the parameters including the steady state a_2 and form their forecasts:

$$E^*_{2t} y_{t+1} = (a_{2x}, a_{2\pi})',$$

where E^*_{2t} is the operator of Type 2 agents' expectations at time t.

The average of both types of forecasts form aggregate expectations in the NK model:

$$\begin{aligned} E^*_t y_{t+1} &= \lambda E^*_{1t} y_{t+1} + (1-\lambda) E^*_{2t} y_{t+1} \\ &= \begin{bmatrix} \lambda x^I + (1-\lambda) a_{2x} \\ \lambda \pi^I + (1-\lambda) a_{2\pi} \end{bmatrix}, \end{aligned} \tag{9}$$

and the actual law of motion (ALM) around the targeted steady state is

$$y_t = \left(A_I + B_I \begin{bmatrix} \lambda x^I + (1-\lambda) a_{2x} \\ \lambda \pi^I + (1-\lambda) a_{2\pi} \end{bmatrix} \right) + C_I w_t,$$

where

$$\begin{aligned} A_I &\equiv \begin{bmatrix} 1 & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha\pi^I (\phi_\pi - 1) \\ 0 \end{bmatrix}, \ B_I \equiv \begin{bmatrix} 1 & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \\ C_I &\equiv \begin{bmatrix} 1 & \alpha\phi_\pi \\ -\kappa & 1 \end{bmatrix}^{-1}. \end{aligned}$$

3.2 Learning

As shocks follow iid processes, parameter a_2 is estimated by the sample mean of y_t, which is represented by a recursive algorithm:

$$a_{2t} = a_{2,t-1} + t^{-1} \left(T(a_{2,t-1}) - a_{2,t-1} \right),$$

where $T(a_2)$ is the map from the PLM to the ALM:

$$T(a_2) \equiv A_I + B_I \begin{bmatrix} \lambda x^I + (1-\lambda) a_{2x} \\ \lambda \pi^I + (1-\lambda) a_{2\pi} \end{bmatrix}.$$

The convergence of the algorithm is governed by the associated ordinary differential equation (ODE):

$$\frac{\mathrm{d}a_2}{\mathrm{d}\tau} = T(a_2) - a_2, \tag{10}$$

where τ denotes the notional time.

Proposition 2 *The fixed point of ODE (10) is equal to the targeted steady state (5).*

Note that the fixed points of the other parameters c_2 are C_I. For the existence of this steady state, Eq. (8) must be satisfied.

At the fixed point, the average of forecast (9) is equal to the targeted steady state. The targeted steady state equilibrium is

$$\begin{aligned} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} x^I \\ \pi^I \end{bmatrix} + C_I w_t, \\ i_t - E_t^* \pi_{t+1} &= r \end{aligned}$$

If the ODE is locally asymptotically stable, parameter a_2 converges to the fixed point under least-squares learning and the economy is determined by the process (10). Under the assumption that shocks follow an iid processes, the other parameter c_2 also converges to their fixed points (see Evans et al., 2008). In this case, the targeted steady state is said to be locally stable under learning.

This steady state exists if the effective lower bound binds (8).

The ODE is locally stable if and only if its Jacobian has all eigenvalues with negative real parts. The stability conditions imposed on the structural parameters are as follows (see Appendix A):

$$\phi_\pi > 1 - \frac{\lambda\left(1 - \beta\left(1 - \lambda\right) + \alpha\kappa\right)}{\alpha\kappa}. \tag{11}$$

We find that the condition imposed on monetary policy parameter ϕ_π is relaxed by an improvement in the credibility of the central bank. In this sense, the credibility of the central bank improves the expectational stability of the targeted steady state. As Eq. (11) is less stringent than the Taylor principle ($\phi_\pi > 1$),

Proposition 3 *If and only if $r + \pi^I - i^U \geq 0$, the targeted steady state (5) exists and is locally stable under learning regardless of the degree of central bank credibility.*

4 Low steady state

Next, we consider the steady state where the nominal interest rate is fixed at the effective lower bound $i_t = i^U$. We examine the stability of the steady state in the presence of imperfectly credible inflation targeting. This situation corresponds to that of many inflation-targeting countries, particularly Japan. This is similar to Arifovic et al. (2018)'s assumption of a perpetual liquidity trap.

4.1 Expectations formation

Suppose that Type 1 agents are informed of the steady states of the inflation rate and output gap from inflation targeting. In this case, agents of Type 1 specify a PLM:

$$y_t = a_1 + c_1 w_t,$$

where $a_1 \equiv \left(x^I, \pi^I\right)'$; that is, agents fix the first element of a_1 at the inflation-output gap target $\left(x^I, \pi^I\right)$. Then, their forecast is

$$E_{1t}^* y_{t+1} = \left(x^I, \pi^I\right)'.$$

Type 2 agents do not believe the inflation target, but specify the following PLM:

$$y_t = a_2 + c_2 w_t,$$

where $a_2 \equiv (a_{2x}, a_{2\pi})'$ and c_2 is a 2×2 matrix of constant terms for w_t and agents must estimate (a_2, c_2). Then, their forecast is

$$E_{2t}^* y_{t+1} = (a_{2x}, a_{2\pi})'.$$

Therefore, the average of both types of forecasts is

$$\begin{aligned} E_t y_{t+1} &= \lambda E_{1t}^* y_{t+1} + (1-\lambda) E_{2t}^* y_{t+1} \\ &= \begin{bmatrix} \lambda x^I + (1-\lambda) a_{2x} \\ \lambda \pi^I + (1-\lambda) a_{2\pi} \end{bmatrix}, \end{aligned}$$

and the ALM on the zero lower bound is

$$y_t = \left(A_U + B_U \begin{bmatrix} \lambda x^I + (1-\lambda) a_{2x} \\ \lambda \pi^I + (1-\lambda) a_{2\pi} \end{bmatrix}\right) + C_U w_t. \tag{12}$$

where

$$A_U \equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} \alpha\left(r - i^U\right) \\ 0 \end{bmatrix}, \quad B_U \equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix},$$

$$C_U \equiv \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}^{-1}.$$

4.2 Learning

As v_t follows an iid process, parameter a_2 is estimated by the sample mean of y_t, which is represented by a recursive algorithm:

$$a_{2t} = a_{2,t-1} + t^{-1}\left(T\left(a_{2,t-1}\right) - a_{2,t-1}\right),$$

where

$$T\left(a_2\right) \equiv A_U + B_U \begin{bmatrix} \lambda x^I + (1-\lambda)\, a_{2x} \\ \lambda \pi^I + (1-\lambda)\, a_{2\pi} \end{bmatrix}.$$

The convergence of the algorithm is governed by the following ODE:

$$\frac{\mathrm{d}a_2}{\mathrm{d}\tau} = T\left(a_2\right) - a_2.$$

The ODE is locally stable if and only if its Jacobian,

$$D\left(T\left(a_2\right) - a_2\right) = (1-\lambda)\, B_U - I_2.$$

has all eigenvalues with negative real parts. Appendix B proves the stability condition:

$$\lambda - (1-\lambda)(\alpha\kappa + \beta\lambda) > 0.$$

Proposition 4 *The low steady state (6) is locally stable under learning if*

$$\lambda > \frac{1}{2\beta}\left(\sqrt{(1-\beta+\alpha\kappa)^2 + 4\alpha\kappa\beta} - (1-\beta+\alpha\kappa)\right) > 0, \tag{13}$$

otherwise, the steady state is locally unstable.

The lower bound of the condition is calibrated as 0.51 for $(\alpha, \kappa, \beta) = (1/1.45, 0.77, 0.99)$, 0.42 for $(4, 0.075, 0.99)$, and 0.20 for $(0.164, 0.3, 0.99)$. This implies that the bound is highly plausible. If a part of agents believe in the inflation target, the low steady state can be stable under learning. If all agents believe the target, the fixed point of a_{1x} does not exist, and the low steady state does not exist. If no agents believe the target, the fixed point does exist, but the stability condition of Type 2 agents is not satisfied.

4.3 Fixed point

The fixed point of the ODE is

$$\begin{aligned}
a_{1x}\left(=E_1^*x_{t+1}\right) &= x^I=\left(\frac{1-\beta}{\kappa}\right)\pi^I,\\
a_{1\pi}\left(=E_1^*\pi_{t+1}\right) &= \pi^I,\\
a_{2x}\left(=E_2^*x_{t+1}\right) &= x^I+\frac{\alpha\left(1-\beta\left(1-\lambda\right)\right)}{\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)}\left(r+\pi^I-i^U\right)\\
&= x^U+\lambda\frac{\alpha\kappa+\left(1-\beta\right)\left(1-\beta\left(1-\lambda\right)\right)}{\kappa\left(\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)\right)}\left(r+\pi^I-i^U\right),\\
a_{2\pi}\left(=E_2^*\pi_{t+1}\right) &= \pi^I+\frac{\alpha\kappa}{\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)}\left(r+\pi^I-i^U\right)\\
&= \pi^U+\lambda\frac{1-\beta\left(1-\lambda\right)+\alpha\kappa}{\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)}\left(r+\pi^I-i^U\right).
\end{aligned}$$

Note that $(a_{2x},a_{2\pi})$ converge to $\left(x^U,\pi^U\right)$ as $\lambda\to 0$, but do not converge to $\left(x^I,\pi^I\right)$ as $\lambda\to 1$.

The steady state forecasts are

$$\begin{aligned}
E_1^*y_{t+1} &= \left(x^I,\pi^I\right)',\\
E_2^*y_{t+1} &= \begin{bmatrix} x^I\\ \pi^I\end{bmatrix}+\frac{\alpha\left(r+\pi^I-i^U\right)}{\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)}\begin{bmatrix}1-\beta\left(1-\lambda\right)\\ \kappa\end{bmatrix}\\
&= \begin{bmatrix} x^U\\ \pi^U\end{bmatrix}+\frac{\lambda\left(r+\pi^I-i^U\right)}{\kappa\left(\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)\right)}\begin{bmatrix}\alpha\kappa+\left(1-\beta\right)\left(1-\beta\left(1-\lambda\right)\right)\\ \kappa\left(\alpha\kappa+1-\beta\left(1-\lambda\right)\right)\end{bmatrix}.
\end{aligned}$$

The average of the forecasts of both types is

$$\begin{aligned}
E^*y_{t+1} &= \begin{bmatrix} x^I\\ \pi^I\end{bmatrix}+\frac{\alpha\left(1-\lambda\right)\left(r+\pi^I-i^U\right)}{\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)}\begin{bmatrix}1-\beta\left(1-\lambda\right)\\ \kappa\end{bmatrix}\\
&= \begin{bmatrix} x^U\\ \pi^U\end{bmatrix}+\frac{\lambda\left(r+\pi^I-i^U\right)}{\kappa\left(\lambda-\left(1-\lambda\right)\left(\alpha\kappa+\beta\lambda\right)\right)}\begin{bmatrix}\left(1-\beta\right)\left(1-\beta\left(1-\lambda\right)\right)+\alpha\kappa\beta\left(1-\lambda\right)\\ \kappa\left(1-\beta\left(1-\lambda\right)\right)\end{bmatrix}.
\end{aligned}$$

At this steady state, $E_1^*i_{t+1}=i^I$ (that is, Type 1 agents do not know i^U and believes that the central bank can achieve i^I (less than i^U) consistent with its inflation target π^I). From $E_2^*i_{t+1}=i^U$, the average of nominal interest rate expectations is:

$$\begin{aligned}
E^*i_{t+1} &= \lambda i^I+\left(1-\lambda\right)i^U\\
&= i^U+\lambda\left(r+\pi^I-i^U\right).
\end{aligned}$$

If $r+\pi^I-i^U\leq 0$,

$$i=i_L>E^*i_{t+1}.$$

This implies that interest rates are expected to fall (i.e., a downward sloping yield curve).

Substituting these results into ALM (12), the steady state equilibrium is obtained as follows:

$$y_t = E^*_{2t} y_{t+1} + C_U w_t.$$

The steady state is equal to Type 2 expectations:

$$y = E^*_2 y_{t+1},$$

and the steady state real interest rate is

$$i - E^* \pi_{t+1} = r - \frac{\lambda \left(1 - \beta \left(1 - \lambda\right)\right)}{\lambda - \left(1 - \lambda\right)\left(\alpha\kappa + \beta\lambda\right)} \left(r + \pi^I - i^U\right).$$

Note that even if all agents form their expectations at the inflation-output target ($\lambda = 1$), the low steady state is not equal to the targeted steady state.

The low steady state exists if the effective lower bound binds (8), that is,

$$i^U > i^I + \phi_\pi \left(\pi - \pi^I\right)$$
$$\left(r + \pi^I - i^U\right) \left(1 + \frac{\alpha\kappa\phi_\pi}{\lambda - \left(1 - \lambda\right)\left(\alpha\kappa + \beta\lambda\right)}\right) < 0.$$

Proposition 5 *Under stability condition (13), the low steady state (6) exists if and only if*

$$i^I = r + \pi^I < i^U. \tag{14}$$

In summary, the low steady state exists under the stability condition (13) and the existence condition (14). This suggests that a long-run liquidity trap always occur when two situations occur simultaneously: first, the nominal interest rate target i^I is less than the effective lower bound i^U; second, the central bank is partly credible enough to ensure the stability of the low steady state.

Proposition 6 *If and only if the stability condition (13) and the existence condition (14) are satisfied, the low steady state exists and is stable under learning and has the following feature:*

$$y = E^*_2 y_{t+1} < E^* y_{t+1} < y^U < E^*_1 y_{t+1} = y^I,$$
$$i - E^* \pi_{t+1} > r.$$

The steady state increases and approaches the targeted steady state y^I as the credibility of the central bank improves.

If $r + \pi^I - i^U < 0$,

$$\begin{aligned} \frac{dE^* y_{t+1}}{d\lambda} &> 0, \\ E^* y_{t+1} &= \begin{cases} y^I & \text{if } \lambda = 1 \\ -\infty & \text{if } \lambda - (1-\lambda)(\alpha\kappa + \beta\lambda) \to +0 \end{cases}, \end{aligned}$$

and

$$\begin{aligned} \frac{dy}{d\lambda} &= \frac{dE_2^* y_{t+1}}{d\lambda} > 0, \\ y &= E_2^* y_{t+1} = \begin{cases} y^U + \left(r + \pi^I - i^U\right) \left[\dfrac{\alpha + \frac{1-\beta}{\kappa}}{\alpha\kappa + 1} \right] & \text{if } \lambda = 1 \\ -\infty \quad \text{if } \lambda - (1-\lambda)(\alpha\kappa + \beta\lambda) \to +0 \end{cases}. \end{aligned}$$

Note that even if $\lambda = 1$, $y = E_2^* y_{t+1}$ does not converge to y^I because the steady state is a liquidity trap.

4.4 Findings

These findings are consistent with empirical results in literature.

4.4.1 Existence of the low steady state

First, we find that if $r + \pi^I - i^U < 0$, the low steady state cannot exist under rational expectations (see Benhabib et al., 2001), but it does exist under learning. This steady state may correspond to the *secular stagnation hypothesis*, defined by Summers (2013, 2014) as a persistently low or negative natural rate of interest leading to a chronically binding zero lower bound (ZLB) (theoretically see Eggertsson et al., 2019). This is a self-confirming equilibrium for Type 2 agents ($y = E_2^* y_{t+1}$). This steady state is more deflationary than that under rational expectations ($y < y^U$). The steady-state nominal interest rate reaches the lower bound ($i = i^U$), but remains greater than the nominal interest rate target ($i = i^U > i^I$). As a result, the steady-state real interest rate $i - E^* \pi_{t+1}$ is greater than the natural rate r. This leads to a more deflationary situation. The real interest rate decreases as the credibility of the central bank improves ($d\left(i - E^* \pi_{t+1}\right)/d\lambda < 0$).

4.4.2 Stable liquidity trap

Second, we find that the low steady state can be expectationally stable if the credibility of the inflation target λ satisfies condition (13). If the central bank is credible for Type 1 agents, their expectations are fixed at the target so that Type 2 agents' expectations formation governs the expectational stability of the low steady state. If the inflation-output target is credible for some agents, the low steady state becomes expectationally stable.

If the low steady state is stable, then a deflationary spiral does not occur. The literature shows that the low steady state is not expectationally stable, so that the economy diverges from the steady state unboundedly. Actually, real economies do not exhibit a deflationary spiral. As a reason for this, Gorodnichenko and Sergeyev (2021) consider the existence of a lower bound on inflation expectations (the zero lower bound on inflation expectations). Instead, our study assumes that people's expectations are fixed by inflation targeting.

4.4.3 Upward bias in inflation expectations

Third, if the low steady state exists and is expectationally stable, the average expectations are upward biased against the steady state, $E^*y_{t+1} > y$. While Type 2 agents form expectations based on available data, Type 1 agents believe that the inflation-output target is the steady state of the economy. Central bank credibility λ increases the proportion of Type 1 agents and the upward bias in inflation expectations $(d\left(E^*y_{t+1} - y\right)/d\lambda > 0)$.

The upward bias is a typical feature of the empirical observations of inflation expectations. Aßhoff et al. (2021) find that using Euro area data, the unconventional monetary policy of the European Central Bank raised short-term inflation expectations, but had no lasting impact on inflation or economic activity in the Euro area. Gorodnichenko and Sergeyev (2021) find that in low-inflation countries (especially Japan), households often form inflation expectations well above the inflation target and less deflationary expectations. Hori and Kawagoe (2013) find an upward bias in Japanese households' inflation expectations in the 2000s. Weber et al. (2022) find evidence that firms' and households' cognitive abilities lead to higher inflation expectations.

Capistrán and Timmermann (2009) provide a preference-based theory that explains biases in inflation expectations by using asymmetries in preferences. Baqaee (2020) depends on ambiguity aversion. Afrouzi and Veldkamp (2020) provide a belief-based explanation that parameter uncertainty over positively skewed distributions leads to a systematic upward bias in people's beliefs about inflation. Gorodnichenko and Sergeyev (2021) derive an upward bias by assuming an exogenous zero lower bound on inflation expectations.

The novelty of our results lies in deriving the upward bias under imperfect central bank credibility in the plausible expectations framework. Recent experience with unconventional monetary policies has cast doubt on the credibility of central banks and their inflation-targeting policies. Adaptive learning models have been confirmed to explain the dynamics of recent inflation persistence and inflation expectations. We show that in this situation, a upward bias in inflation expectations can occur during periods of negative natural rates. This finding is consistent with the upward bias observed after the global financial crisis and during Japan's past three decades.

4.4.4 Heterogeneous expectations

Fourth, the heterogeneity in expectations persists at a steady state. For the targeted steady state, even if Type 2 agents do not believe the inflation target, their expectations can converge to the targeted steady state in the presence of Type 1 agents believing the target. On the other hand, for the low steady state, even if Type 1 expects the realization of the targeted steady state in the future, Type 2's expectations converge to the low steady state. Heterogeneity in expectations decreases in central bank credibility $\left(d\left(E_1^* y_{t+1} - E_2^* y_{t+1}\right)/d\lambda < 0\right)$. However, a liquidity trap with imperfectly credible inflation targeting holds heterogeneity under learning.

Long-term heterogeneity in expectations is observed in deflationary countries. Hori and Kawagoe (2013) find similar heterogeneity among Japanese households. Hattori and Yetman (2017) confirm that the dispersion of inflation expectations among Japanese professional forecasters is larger than in the U.S. and Canada. Diamond et al. (2020) focus on the deflationary period since 1995 and find that Japanese households' inflation expectations differ from each other based on their individual inflation experiences. Our results are consistent with these empirical findings.

5 Simulation

We illustrate the stability of the low steady state under inflation targeting using real-time simulations. We set deep parameters as $(\alpha, \beta, \kappa) = (1/1.45, 0.99, 0.77)$, $\phi_\pi = 1.5$, $\pi^I = 0.5$, $r = -1.0$, $\lambda = 0.7$.

Figure 3 illustrates the instability of the equilibrium without inflation targeting. We simulate the updating of the parameter estimates (a, c) in recursive least-squares estimations and the corresponding paths for the temporary equilibrium (x_t, π_t). The initial values for the parameters and equilibria are set at the fixed points, each of which is indicated by the horizontal solid line in each panel. A decreasing-gain algorithm is used in the simulation.[3] The shock v_t follows $N(0,1)$.

As previously clarified, the low steady-state equilibrium is unstable in the absence of inflation targeting. The constant terms of Type 2 $a_2 = (a_{2x}, a_{2\pi})'$ diverge from the fixed points and explode with equilibrium paths. This explosion then destabilizes the updating of average $a = (a_x, a_\pi)$ and coefficients c, leading to further economic fluctuations.

Figure 4 shows the corresponding simulation in the presence of inflation targeting, under which parameter $a_1 = (a_{1x}, a_{1\pi})$ is fixed at the inflation-output target $\left(x^I, \pi^I\right)$. We see that inflation targeting stabilizes agents' expectations formation. The constant term parameters of Type 2 and the average a converge to the fixed point. This leads to fast convergence of the other parameter estimates, making the economy stable around the steady state.

6 Discussion

In light of previous results, what possible solutions can be considered for Japan's liquidity trap? One possible solution may be to raise the inflation target. Proposition 5 suggests that if π^I is as high as to violate the condition (14), there will be no stable low steady state. In an economy with a low natural rate of interest r, the inflation target should be raised by that amount so that the targeted nominal interest rate i^I exceeds its lower bound i^U. This ensures that the intended steady state is uniquely stable.

[3] Following Evans and McGough (2020), we use a decreasing gain sequence with $\gamma_t = t^{-0.8}$ rather than $\gamma_t = t^{-1}$ to increase the speed of convergence.

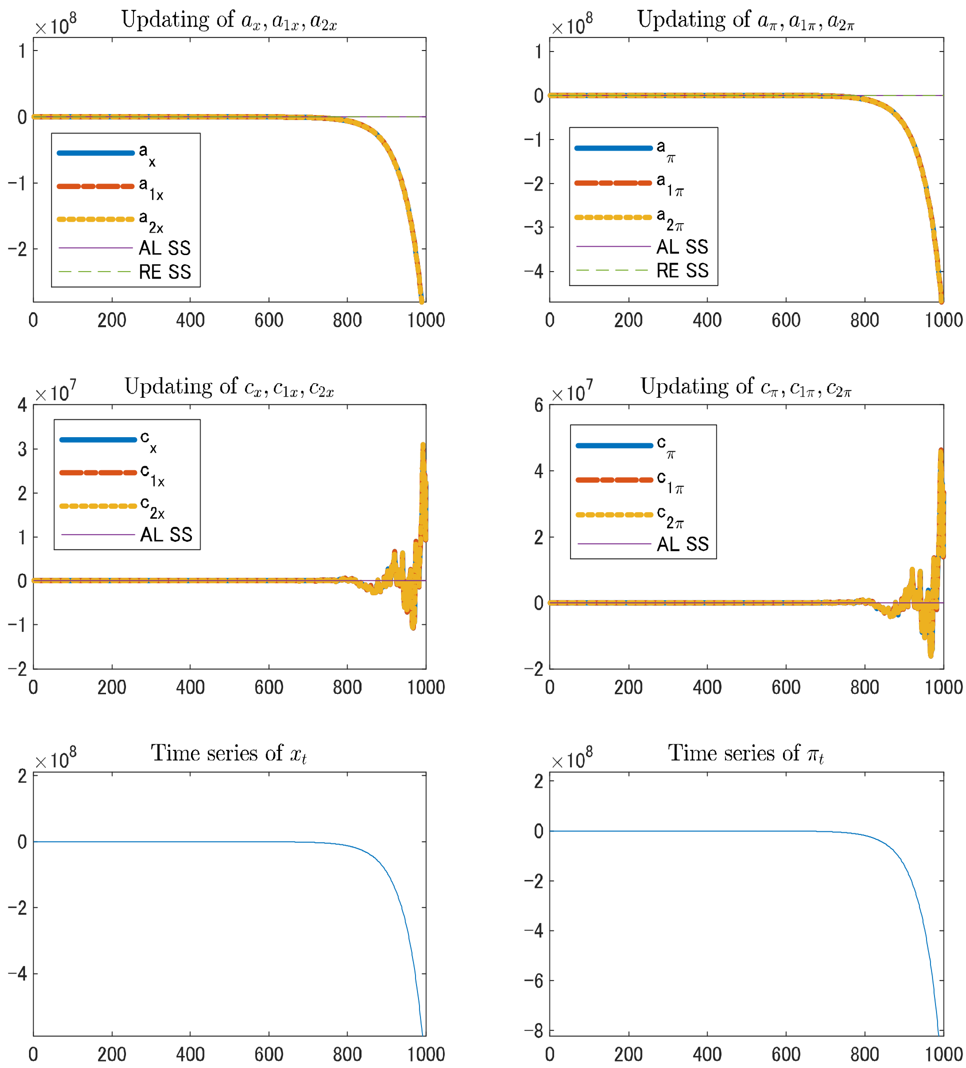

Figure 3: Simulation without inflation targeting for (a, c) (top six panels) and (x, π) (bottom two panels). "AL SS" and "RE SS" represent the fixed point under adaptive learning and the steady state under rational expectations, repsectively. $(\alpha, \beta, \kappa) = (1/1.45, 0.99, 0.77)$. $\phi_\pi = 1.5$, $\pi^I = 0.5$, $r = -1.0$, $\lambda = 0.7$.

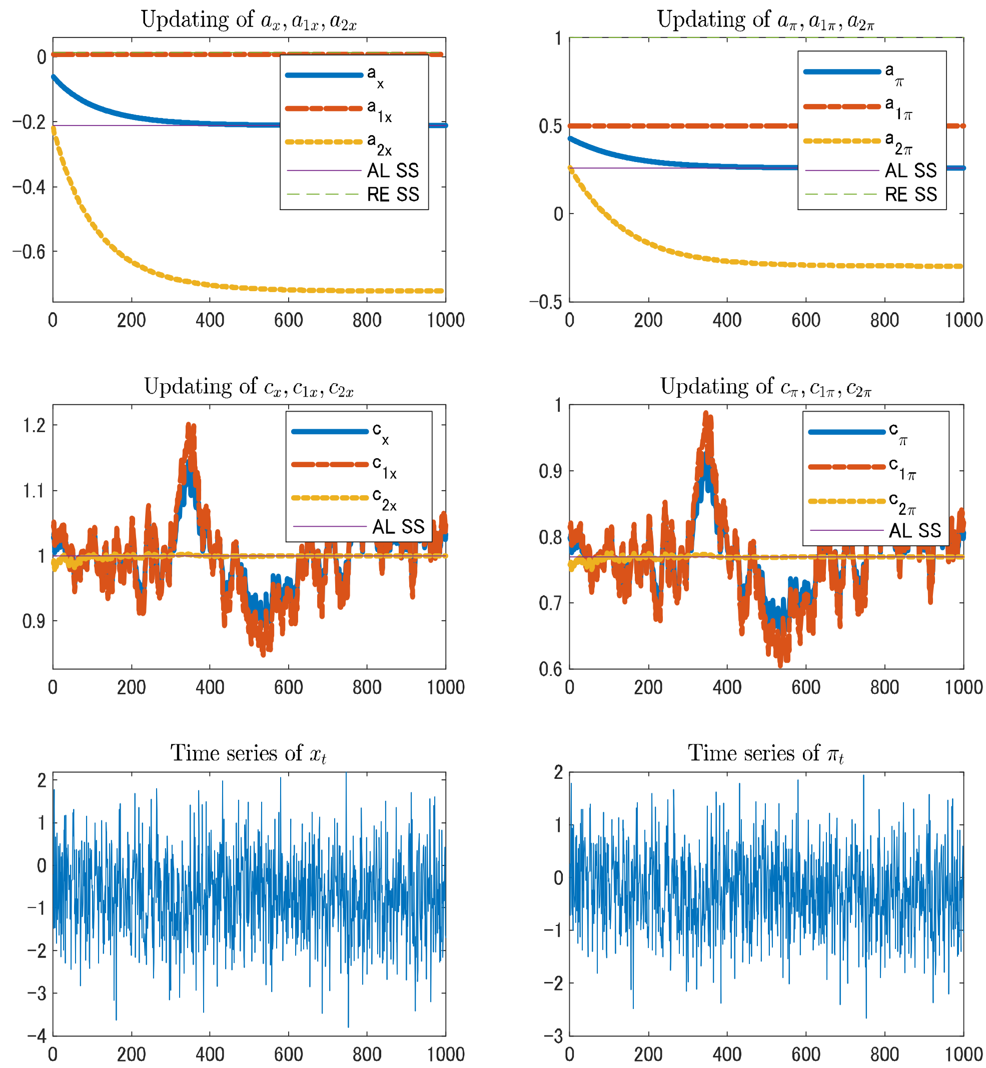

Figure 4: Simulation with inflation targeting for (a, c) (top six panels) and (x, π) (bottom two panels). "AL SS" and "RE SS" represent the fixed point under adaptive learning and the steady state under rational expectations, respectively. $(\alpha, \beta, \kappa) = (1/1.45, 0.99, 0.77)$. $\phi_\pi = 1.5$, $\pi^I = 0.5$, $r = -1.0$, $\lambda = 0.7$.

Another possible solution is to make agents' expectations formation flexible. Equation (13) indicates that the low steady state is stable if many agents anchor their expectations to the inflation target. Conversely, if agents deanchor and update their expectations each period, the low steady state should be unstable, making the intended steady state uniquely stable.

The recent upward trend in Japanese inflation can be explained by the deanchoring of people's expectations. This may have been triggered by rising oil prices and the depreciation of the yen, which may have led people to believe that prices can fluctuate. Even if these fluctuations are temporary, the deanchoring of expectations may help the Japanese economy to leave the liquidity trap and converge to the intended steady state.

On the other hand, since the Bank of Japan (BOJ) introduced inflation targeting in 2013, no clear inflation was realized. The BOJ has sought to raise people's inflation expectations by introducing "forward guidance," "quantitative and qualitative easing," and "yield curve control." However, market interventions have been exclusively through government bonds, with only a few interventions through risk assets. In contrast, since the global financial crisis of the late 2000s, the Federal Reserve has actively intervened not only through government bonds but also through risk assets such as mortgage-backed securities. While it would be prudent to exercise caution in implementing similar policies immediately in Japan, it would nevertheless be beneficial to consider them in order to bring an end to a quarter of a century of the liquidity trap.

7 Conclusion

This paper has re-examined the stability of a liquidity trap under adaptive learning. We assume that the central bank announces an inflation target that anchors agents' expectations at the target. We examine whether the liquidity trap steady state is expectationally stable in the presence of inflation targeting. We also consider the case where the central bank (or inflation targeting) is imperfectly credible such that agents' expectations are imperfectly anchored. We then examine how the stability of the liquidity trap is affected by the degree of central bank credibility. In addition, we clarify the characteristics of the liquidity trap.

This paper finds that the liquidity trap can be expectationally stable in the presence of inflation targeting. The targeted steady state is always stable under learning regardless of the degree of central bank credibility, while the low steady state becomes stable if the central bank is highly credible and the natural rate of interest is negative. In the stable liquidity trap, the economy exhibits an upward bias in inflation expectations and heterogeneous inflation expectations. These findings are consistent with empirical evidence.

Future research will examine the stability of a liquidity trap in a situation where agents fix their long-run inflation expectations at the level of the liquidity trap. It is argued that Japan's deflationary experience of more than two decades has created a "social norm" that prices will not rise. We need to clarify whether such a fixed expectations formation leads the economy into a long-run liquidity trap.

Appendix

A Stability condition of the targeted steady state

The Jacobian is,

$$
\begin{aligned}
& D\left(T\left(a_2\right)-\left(a_2\right)\right) \\
= \; & \frac{1}{\alpha\kappa\phi_\pi+1}\begin{bmatrix} \lambda-\left(\alpha\kappa\phi_\pi+1\right) & 1-\lambda & \alpha\left(1-\lambda\right)\left(1-\beta\phi_\pi\right) \\ \lambda & 1-\lambda-\left(\alpha\kappa\phi_\pi+1\right) & \alpha\left(1-\lambda\right)\left(1-\beta\phi_\pi\right) \\ \kappa\lambda & \kappa\left(1-\lambda\right) & \left(1-\lambda\right)\left(\beta+\alpha\kappa\right)-\left(\alpha\kappa\phi_\pi+1\right) \end{bmatrix}.
\end{aligned}
$$

The Routh-Hurwitz stability criterion suggests that the ODE is stable if and only if the Jacobian has a characteristic polynomial,

$$\det\left(\gamma I-A\right)=\gamma^2+\alpha_1\gamma+\alpha_0,$$

in which γ is either of the eigenvalue of the Jacobian and the coefficients satisfy

$$\alpha_0>0, \quad \alpha_1>0.$$

These provide

$$
\begin{aligned}
\phi_\pi &> 1-\frac{\lambda\left(1-\beta\left(1-\lambda\right)+\alpha\kappa\right)}{\alpha\kappa}, \\
\phi_\pi &> 1-\frac{\lambda\left(1-\beta\left(1-\lambda\right)+\alpha\kappa\right)}{\alpha\kappa}-\frac{\left(1-\lambda\right)\left(1-\beta+\alpha\kappa+2\beta\lambda\right)}{2\alpha\kappa}.
\end{aligned}
$$

The second condition is redundant, and the first condition is a unique sufficient and necessary condition for the stability of the ODE. This condition is more relaxed than $\phi_\pi + \lambda > 1$ (that is, the condition under the announcement of only the inflation target π^I).

B Stability condition of the low steady state

The Jacobian is,

$$D\left(T\left(a_2\right) - a_2\right) = \left(1-\lambda\right) B_U - I_2.$$

The Routh-Hurwitz stability criterion suggests that the ODE is stable if and only if the Jacobian has a characteristic polynomial,

$$\det\left(\gamma I - A\right) = \gamma^2 + \alpha_1 \gamma + \alpha_0,$$

in which γ is either of the eigenvalue of the Jacobian and the coefficients satisfy

$$\alpha_0 > 0, \quad \alpha_1 > 0.$$

That is,

$$\begin{aligned} \lambda - \left(1-\lambda\right)\left(\alpha\kappa + \beta\lambda\right) &> 0, \\ \lambda - \frac{\beta + \alpha\kappa - 1}{\beta + \alpha\kappa + 1} &> 0. \end{aligned}$$

These conditions are simplified as follows:

$$\lambda > \frac{1}{2\beta}\left(\sqrt{\left(1-\beta+\alpha\kappa\right)^2 + 4\alpha\kappa\beta} - \left(1-\beta+\alpha\kappa\right)\right) > 0,$$

which is the unique sufficient and necessary condition of the stability of the ODE.

References

Afrouzi, H. and L. Veldkamp (2020): "Biased Inflation Forecasts," *mimeo*, 1–22.

Arifovic, J., S. Schmitt-Grohé, and M. Uribe (2018): "Learning to live in a liquidity trap," *Journal of Economic Dynamics and Control*, 89, 120–136.

Aruoba, S. B., P. Cuba-Borda, and F. Schorfheide (2018): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries," *Review of Economic Studies*, 85, 87–118.

Asshoff, S., A. Belke, and T. Osowski (2021): "Unconventional monetary policy and inflation expectations in the Euro area," *Economic Modelling*, 102, 105564.

Baqaee, D. R. (2020): "Asymmetric inflation expectations, downward rigidity of wages, and asymmetric business cycles," *Journal of Monetary Economics*, 114, 174–193.

Benhabib, J., S. Schmitt-Grohé, and M. Uribe (2001): "The Perils of Taylor Rules," *Journal of Economic Theory*, 96, 40–69.

Bernanke, B. S., T. Laubach, F. S. Mishkin, and A. S. Posen (1999): *Inflation Targeting: Lessons from the International Experience*, Princeton University Press.

Bilbiie, F. O. (2022): "Neo-Fisherian Policies and Liquidity Traps," *American Economic Journal: Macroeconomics*, 14, 378–403.

Brand, C., M. Bielecki, and A. Penalver (2018): "The natural rate of interest: estimates, drivers, and challenges to monetary policy," *ECB Occasional Paper*.

Bullard, J. (2010): "Seven Faces of "The Peril"," *Federal Reserve Bank of St. Louis Review*, 92, 339–352.

Candia, B., O. Coibion, and Y. Gorodnichenko (2022): "The Macroeconomic Expectations of Firms," *NBER Working Paper Series*, 30042.

Candia, B., M. Weber, Y. Gorodnichenko, and O. Coibion (2023): "Perceived and Expected Rates of Inflation of US Firms," *AEA Papers and Proceedings*, 113, 52–55.

Capistrán, C. and A. Timmermann (2009): "Disagreement and Biases in Inflation Expectations," *Journal of Money, Credit and Banking*, 41, 365–396.

Carvalho, C., S. Eusepi, E. Moench, and B. Preston (2023): "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, 15, 1–47.

Cochrane, J. H. (2018): "Michelson-Morley, Fisher, and Occam: The Radical Implications of Stable Quiet Inflation at the Zero Bound," *NBER Macroeconomics Annual*, 32, 113–226.

Crump, R. K., S. Eusepi, E. Moench, and B. Preston (2023): "The term structure of expectations," in *Handbook of Economic Expectations*, ed. by R. Bachmann, G. Topa, and W. van der Klaauw, Academic Press, chap. 17, 507–540.

Diamond, J., K. Watanabe, and T. Watanabe (2020): "THE FORMATION OF CONSUMER INFLATION EXPECTATIONS: NEW EVIDENCE FROM JAPAN'S DEFLATION EXPERIENCE," *International Economic Review*, 61, 241–281.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019): "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, 11, 1–48.

Eggertsson, G. B. and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, 139–233.

Erceg, C. J. and A. T. Levin (2003): "Imperfect credibility and inflation persistence," *Journal of Monetary Economics*, 50, 915–944.

Eusepi, S. (2007): "Learnability and monetary policy: A global perspective," *Journal of Monetary Economics*, 54, 1115–1131.

——— (2010): "Central Bank Communication and the Liquidity Trap," *Journal of Money, Credit and Banking*, 42, 373–397.

Eusepi, S. and B. Preston (2011): "Expectations, Learning, and Business Cycle Fluctuations," *American Economic Review*, 101, 2844–2872.

Evans, G. W., E. Guse, and S. Honkapohja (2008): "Liquidity traps, learning and stagnation," *European Economic Review*, 52, 1438–1463.

Evans, G. W. and S. Honkapohja (2005): "Policy interaction, expectations and the liquidity trap," *Review of Economic Dynamics*, 8, 303–323.

Evans, G. W. and B. McGough (2018): "Interest-Rate Pegs in New Keynesian Models," *Journal of Money, Credit and Banking*, 50, 939–965.

——— (2020): "Stable near-rational sunspot equilibria," *Journal of Economic Theory*, 186, 104987.

Gorodnichenko, Y. and D. Sergeyev (2021): "Zero Lower Bound on Inflation Expectations," *NBER Working Papers Series*, 29496.

Hattori, M. and J. Yetman (2017): "The evolution of inflation expectations in Japan," *Journal of the Japanese and International Economies*, 46, 53–68.

Ho, T.-k., Y.-c. Lin, and K.-c. Yeh (2021): "Alternative Monetary Policies Under Keynesian Animal Spirits," *Macroeconomic Dynamics*, 25, 213–239.

Hommes, C. and J. Lustenhouwer (2019a): "Inflation targeting and liquidity traps under endogenous credibility," *Journal of Monetary Economics*, 107, 48–62.

——— (2019b): "Managing unanchored, heterogeneous expectations and liquidity traps," *Journal of Economic Dynamics and Control*, 101, 1–16.

Hori, M. and M. Kawagoe (2013): "Inflation expectations of Japanese households: Micro evidence from a consumer confidence survey," *Hitotsubashi Journal of Economics*, 54, 17–38.

Krugman, P. (2000): "Thinking About the Liquidity Trap," *Journal of the Japanese and International Economies*, 14, 221–237.

Krugman, P. R. (1998): "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 1998, 137.

Lustenhouwer, J. (2021): "Unanchored Expectations: Self-reinforcing Liquidity Traps and Multiple Steady States," *Macroeconomic Dynamics*, 25, 845–873.

Malmendier, U. and S. Nagel (2016): "Learning from Inflation Experiences," *Quarterly Journal of Economics*, 131, 53–87.

McCallum, B. (2001): "Inflation Targeting and the Liquidity Trap," Tech. rep., National Bureau of Economic Research, Cambridge, MA.

Mehrotra, A. and J. Yetman (2018): "Decaying Expectations: What Inflation Forecasts Tell Us about the Anchoring of Inflation Expectations," *International Journal of Central Banking*, 14, 55–101.

Mertens, K. R. S. M. and M. O. Ravn (2014): "Fiscal Policy in an Expectations-Driven Liquidity Trap," *Review of Economic Studies*, 81, 1637–1667.

Nakagawa, R. (2022): "Inflation targeting, expectations formation, and macroeconomic stability," *RISS Discussion Paper Series*, 104.

Okazaki, Y. and N. Sudo (2018): "Natural Rate of Interest in Japan – Measuring its size and identifying drivers based on a DSGE model –," *Bank of Japan Working Paper Series*.

Orphanides, A. and J. C. Williams (2005): "Inflation scares and forecast-based monetary policy," *Review of Economic Dynamics*, 8, 498–527.

Schmitt-Grohé, S. and M. Uribe (2017): "Liquidity Traps and Jobless Recoveries," *American Economic Journal: Macroeconomics*, 9, 165–204.

Summers, L. H. (2013): "Why Stagnation Might Prove to be the New Normal," .

——— (2014): "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound," *Business Economics*, 49, 65–73.

——— (2015): "Have we Entered an Age of Secular Stagnation? IMF Fourteenth Annual Research Conference in Honor of Stanley Fischer, Washington, DC," *IMF Economic Review*, 63, 277–280.

——— (2016): "The age of secular stagnation: What it is and what to do about it," *Foreign affairs*, 95, 2–9.

Uribe, M. (2022): "The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models," *American Economic Journal: Macroeconomics*, 14, 133–162.

Veirman, E. D. (2009): "What Makes the Output-Inflation Trade-Off Change? The Absence of Accelerating Deflation in Japan," *Journal of Money, Credit and Banking*, 41, 1117–1140.

Weber, M., F. D'Acunto, Y. Gorodnichenko, and O. Coibion (2022): "The Subjective Inflation Expectations of Households and Firms: Measurement, Determinants, and Implications," *The Journal of Economic Perspectives*, 36, 157–184.